**A4** SIMPLIFY AND MANIPULATE ALGEBRAIC EXPRESSIONS (INCLUDING THOSE INVOLVING SURDS **AND ALGEBRAIC FRACTIONS**) BY EXPANDING PRODUCTSOF TWO **OR MORE** BINOMIALS **(higher tier)**

You should already be able to expand algebraic expressions of the form (*ax + b*)(*cx + d*).

e.g. (2*x* + 1)(3*x* − 2) = 6*x*2 − 4*x* + 3*x* − 2 = 6*x*2 − *x* − 2

e.g. (5*x* + 4)(5*x* − 4) = 25*x*2 − 20*x* + 20*x* − 16 = 25*x*2 − 16

We are now going to algebraic expressions of the form (*ax + b*)(*cx + d*)(*ex + f*).

**EXAMPLE 1**

Expand and simplify (*x* − 2)(2*x* + 3)(*x* + 7)

(*x* − 2)(2*x* + 3) = 2*x*2 + 3*x* − 4*x* − 6 First expand two of the brackets

(You may prefer to use the grid method)

= 2*x*2 − *x* − 6 Simplify

Now (*x* − 2)(2*x* + 3)(*x* + 7) = (*x* + 7)(2*x*2 − *x* − 6)

= *x*(2*x*2 − *x* − 6) + 7(2*x*2 − *x* − 6) Multiply your expansion by each term

in the 3rd bracket

= 2*x*3 − *x*2 − 6*x* + 14*x*2 − 7*x* − 42

= 2*x*3 + 13*x*2 − 13*x* − 42 Simplify

**EXAMPLE 2**

Show that (2*x* + 5)( *x* − 1)(4*x* − 3) = 8*x*3 + 6*x*2 − 29*x* + 15 for all values of *x*.

(2*x* + 5)(*x* − 1) = 2*x*2 − 2*x* + 5*x* − 5 First expand any two of the brackets.   
 = 2*x*2 + 3*x* − 5 Simplify

Now (2*x* + 5)(*x* − 1)(4*x* − 3) = (4*x* − 3)( 2*x*2 + 3*x* − 5)

= 4*x*(2*x*2 + 3*x* − 5) − 3(2*x*2 + 3*x* − 5) Multiply your expansion by each term

in the 3rd bracket

= 8*x*3 + 12*x*2 − 20*x* − 6*x*2 − 9*x* + 15 Remember the minus outside the 2nd bracket

changes each sign inside the 2nd bracket

= 8*x*3 + 6*x*2 − 29*x* + 15 Simplify

To simplify the product of three binomials, first expand any two of the brackets and then multiply this answer by each term in the third bracket.

**EXAMPLE 3**

Expand and simplify (*x* + 3)(*x* − 3)(2*x* + 1)(5*x* − 6)

(*x* + 3)(*x* − 3) = *x*2 − 9 Expand two of the brackets

(2*x* + 1)(5*x* − 6) = 10*x*2 − 7*x* − 6 Expand the other two brackets

(*x* + 3)(*x* − 3)(2*x* + 1)(5*x* − 6)

= (*x*2 − 9)(10*x*2 − 7*x* − 6) Use the two expansions above

= *x*2(10*x*2 − 7*x* − 6) − 9(10*x*2 − 7*x* − 6) Multiply the 2nd bracket

by each term in the 1st bracket

= 10*x*4 − 7*x*3 − 6*x*2 − 90*x*2 + 63*x* + 54

= 10*x*4 − 7*x*3 − 96*x*2 + 63*x* + 54 Simplify

To simplify the product of four binomials, first expand any two of the brackets and then expand the other two brackets and then multiply these answers.

**EXERCISE:**

1. Expand and simplify

(a) (*x* + 1)(*x* + 4)(*x* + 5) (b) (2*x* + 7)(3*x* + 1)(*x* + 8)

(c) (*x* − 3)(*x* − 1)(2*x* − 3) (d) (3*x* + 8)(*x* − 2)(2*x* − 5)

(e) (5*x* − 1)(2*x* + 5)(3*x* − 2) (f) (4*x* + 1)(2*x* + 7)(4*x* − 1)

(g) (*x* + 4)2 (3*x* − 7) (h) (6*x* + 5)(2*x* − 1)2

(i) (*x* − 1)(*x* + 1)(4*x* − 1)(2*x* − 5) (j) (*x* + 5)2(*x* − 2)2

2. Show that (2*x* + 3)3 = 8*x*3 + 36*x*2 + 54*x* + 27 for all values of *x*.

3. Show that (*x* − 4)2(*x* + 3) simplifies to *x*3 + *ax*2 + *bx + c* where *a*, *b* and *c* are integers.

4. Express (3*x* − 1)4 in the form *ax*4 + *bx*3 + *cx*2 + *dx + e* where *a*, *b*, *c*, *d* and *e* are integers.

5. (3*x* + 5)(*x* − 4)(3*x* − 2) = 9*x*3 + *Ax*2 + *Bx* + 40

Work out the value of *A* and the value of *B*.

6. (*x* − 3)(2*x* + 1)(*Ax* + 1) = 8*x*3 + *Bx*2 + *Cx* − 3

Work out the value of *A*, the value of *B* and the value of *C*.

7. Here is a cuboid.

*x* − 2

*x* + 5

2*x* + 1

All measurements are in centimetres.

Show that the volume of the cuboid is (2*x*3 + 7*x*2 − 17*x* − 10) cm3.

8. f(*x*) = 3*x*3 − 2*x*2 + 4

Express f(*x* + 2) in the form *ax*3 + *bx*2 + *cx + d*.

9. The smallest of three consecutive positive odd numbers is (2*x* − 1).

Work out the product of the three numbers.

Give your answer in the form *ax*3 + *bx*2 + *cx + d*.

**ANSWERS:**

1. (a) *x*3 + 10*x*2 + 29*x* + 20 (b) 6*x*3 + 71*x*2 + 191*x* + 56

(c) 2*x*3 − 11*x*2 + 18*x* − 9 (d) 6*x*3 − 11*x*2 − 42*x* + 80

(e) 30*x*3 + 49*x*2 − 61*x* + 10 (f) 32*x*3 + 112*x*2 − 2*x* − 7

(g) 3*x*3 + 17*x*2 − 8*x* − 112 (h) 24*x*3 − 4*x*2 − 14*x* + 5

(i) 8*x*4 − 22*x*3 − 3*x*2 + 22*x* − 5 (j) *x*4 + 6*x*3 − 11*x*2 − 60*x* + 100

2. Proof

3. *x*3 − 5*x*2 − 8*x +* 48

4. 81*x*4 − 108*x*3 + 54*x*2 − 12*x +* 1

5. *A* = −27 *B* = −46

6. *A* = 4 *B* = −18 *C = −*17.

7. Proof

8. 3*x*3 + 16*x*2 + 28*x +* 20

9. 8*x*3 + 12*x*2 − 2*x −* 3